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# DYNAMICS OF INERT GAS BUBBLES IN FORCED CONVECTIVE SYSTEMS

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**Abstract** — This work is an extension of the previously published work dealing with the effects of mass, momentum and thermal variations on the dynamics of vapor–gas bubbles moving through a dilute two-component solution of the gas in a liquid and subjected to various liquid pressure and temperature transients. In particular, the cases of a bubble flowing with a liquid through a channel with a linear temperature rise and pressure drop (a normal heated channel), a channel with a linear temperature drop (a heat exchanger), and a channel experiencing a sudden pressure drop with an increasing temperature (a 'blocked' heated channel) are all being studied. Specific numerical results were obtained for several types of inert gases in liquid sodium for the case of a normal heated channel.

## NOMENCLATURE

$C_A$, mass concentration of inert gas in the solution;
$C_{A0}$, value of $C_A$ far away from the bubble and given by equation (15);
$C_{As}$, saturation value of $C_A$ at bubble interface;
$C_p$, constant pressure specific heat of liquid–gas solution;
$\mathscr{D}_{AB}$, diffusion coefficient of inert gas in liquid–gas solution;
$D_H$, hydraulic diameter of heated channel;
$D_{Hx}$, hydraulic diameter of heat exchanger;
$J$, integral defined in equation (13);
$K$, dimensionless Henry's coefficient, $K_H[T(t)]/K_H(T_\infty)$;
$K_H(T)$, Henry's coefficient;
$L$, length of heated channel;
$m_A$, mass of inert gas in bubble;
$M_A$, molecular weight of inert gas;
$M_B$, molecular weight of liquid;
$p_A$, partial pressure of inert gas inside the bubble;
$p_{A\infty}$, partial pressure of inert gas in the gas blanket;
$p_{Hx}$, constant liquid pressure inside heat exchanger;
$p_L$, liquid pressure;
$p_{L,b}$, liquid pressure after flow blockage;
$p_v$, partial pressure of vapor inside the bubble;
$p_{L0}$, initial liquid pressure, $p_L(0)$;
$q_{Hx}$, heat exchanger heat flux;
$q_w$, heated channel heat flux;
$r$, radial coordinate;
$R$, bubble radius;
$Re$, liquid Reynolds number, $uD_H/\nu$;
$R_0$, initial bubble radius, $R(0)$;
$R'$, universal gas constant;
$S$, dimensionless surface tension, $2\sigma[T(t)]/p_{A\infty}$;
$t$, time;
$t_B$, transit time before the flow blockage, $L/u$;
$T$, temperature inside the bubble ($\equiv T_L$);
$T_{Hx,0}$, heat exchanger inlet temperature;
$T_L$, liquid temperature;
$T_0$, initial liquid temperature, $T_L(0)$;
$T_\infty$, temperature of plenum and gas blanket;
$u$, liquid velocity;
$x$, axial coordinate;
$x_A$, mole fraction of inert gas.

Greek symbols

$\beta$, dimensionless parameter, $R_0^2 p_{A\infty}/\rho_B \mathscr{D}_{AB}^2$;
$\gamma$, parameter, $(R'R_0/M_A)\sqrt{(\mathscr{D}_{AB}/\pi)}$;
$\gamma_{Hx,T}$, dimensionless parameter, $4q_{Hx}R'R_0^2K_H(T_\infty)/M_Bc_pD_{Hx}\mathscr{D}_{AB}$;
$\gamma_p$, dimensionless parameter, $u^3R_0^2\lambda\rho_B/2\mathscr{D}_{AB}p_{A\infty}D_H$;
$\gamma_T$, dimensionless parameter, $4q_wR'R_0^2K_H(T_\infty)/M_Bc_pD_H\mathscr{D}_{AB}$;
$\Delta p$, liquid pressure drop across heated channel;
$\eta$, dimensionless radial coordinate, $(r/R) - 1$;
$\theta$, dimensionless liquid temperature, $[\rho_BR'K_H(T_\infty)/M_B]T(t)$;
$\theta_{Hx,0}$, dimensionless heat exchanger inlet temperature, $[\rho_BR'K_H(T_\infty)/M_B]T_{Hx,0}$;
$\theta_0$, dimensionless heating channel inlet temperature, $[\rho_BR'K_H(T_\infty)/M_B]T_0$;
$\lambda$, friction factor $= 0.0032 + 0.221/Re^{0.237}$;
$\nu$, liquid kinematic viscosity;
$\Pi$, dimensionless inert gas pressure, $p_A(t)/p_{A\infty}$;

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$\Pi_{Hx}$, dimensionless heat exchanger pressure, $p_{Hx}/p_{A\infty}$;
$\Pi_L$, dimensionless liquid pressure, $p_L(t)/p_{A\infty}$;
$\Pi_{L,b}$, dimensionless liquid pressure after flow blockage, $p_{L,b}/p_{A\infty}$;
$\Pi_{L,0}$, dimensionless initial liquid pressure, $p_{L0}/p_{A\infty}$;
$\Pi_v$, dimensionless vapor pressure, $p_v[T(t)]/p_{A\infty}$;
$\rho, \rho_B$, liquid density;
$\sigma$, surface tension;
$\tau$, dimensionless time, $(\mathscr{D}_{AB}/R_0^2)L/u$;
$\tau_B$, dimensionless transit time before the flow blockage;
$\phi$, dimensionless concentration function, $(r/RC_{A0})[C_A(r,t) - C_{A0}]$;
$\phi_s$, dimensionless saturated concentration function, $[C_{As}(t) - C_{A0}]/C_{A0}$;
$\Omega$, dimensionless bubble radius, $R(t)/R_0$.

Subscripts

$A$, inert gas;
0, initial or inlet value;
$s$, saturation value;
$\infty$, in the plenum or gas blanket.

## INTRODUCTION

THE RATE of growth or collapse of a bubble of vapor in its liquid has long been an important engineering subject receiving considerable analytical and experimental attention. The practical applications of the results of this effort are numerous in such areas as boiling and condensing fluids. Mathematical treatments of the situation with varying degrees of completeness, and applied to a wide variety of specialized problems, date back to the early work of Rayleigh [1], and refinements are in current progress, for example, the recent work of Jones and Zuber [2]. The mathematical sophistication employed in the various analyses was often dictated by the forcing functions of the problems studied and by the magnitude of the time-rate-of-change of those functions.

An important complication to the mathematical prediction of bubble dynamics is introduced when a noncondensible inert gas is present in fluid. The analytical treatment of one class of such problems by Epstein and Plesset [3] is widely regarded as classical. This solution neglects the importance of several physical phenomena, and as a result, it applies to certain problems wherein the rate-of-change of the forcing function (the rate of the transient) is relatively small.

Recently, the need to analyze certain postulated off-normal operating conditions and consequences in a nuclear electric-power reactor prompted the development of an analysis that is applicable to a variety of situations in the fast-transient class [4–7]. The physical problem treated in this analysis was a bubble of inert gas and vapor in a solution of liquid and inert gas. The bubble growth or collapse was governed by the time-rate-of-change of parameters in the liquid-gas solution. These parameter transients were the forcing functions for the problem and their rates were relatively fast. The following physical phenomena were modelled in the analysis of the problem

(a) mass transfer of inert gas through the bubble boundary;
(b) compressibility of mass inside the bubble;
(c) diffusion of inert gas in the liquid-gas solution;
(d) inertia of liquid-gas solution;
(e) viscosity of liquid-gas solution; and
(f) surface tension at the bubble boundary.

The inclusion of all of these effects led to a set of second-order nonlinear ordinary differential equations. The solution yielded predictions of the bubble growth and collapse rates, gas pressure, and bubble mass under conditions varying from quasi-steady to fast transient conditions (e.g. 3300 °C s$^{-1}$). The extensive nature of the mathematical formulation made it equally applicable to inertia-controlled and heat-transfer-controlled growth regions as well as to situations not dominated by either of these effects. However, as in most analyses of this nature, some assumptions and exclusions were still incorporated into the model, or allowed by the particular problem for which a solution was sought.

The analysis described in [4, 5] has three major items included in it which limit its generality and thus restricts the application of this relatively extensive work. Two of these items are related to the treatment of the species continuity equation for inert gas in the liquid-gas solution which govern the diffusion of gas in that solution. First, the convection of inert gas in the liquid-gas solution was neglected, which reduced the species continuity equation to a linear second-order partial differential equation. Second, a closed-form solution was obtained to the resulting equation which neglected an integral term that was later shown to be of some importance in certain situations [8]. These two assumptions have generally been incorporated into prior less extensive analyses, although the basis for them is based more on physical arguments than on mathematical comparisons of the terms in the governing equation.

The third major assumption employed in the analysis of [4, 5] was related to the importance of the temperature gradient in the liquid-gas solution. It was assumed that neglecting the gradient would have a small effect on the results, and consequently, a very simple form of the energy equation was employed.

It is the objective of the current work to extend the analysis of [4, 5] to make it applicable to a much larger class of problems. This objective will be accomplished by replacing the three major assumptions discussed above by more vigorous mathematical models.

The extension will be done in three parts. This first part involves the inclusion of the integral term previously neglected. The resulting differential equations remain ordinary, nonlinear, and of second-order. The method of solution previously used [4, 5] still applies and new predictions were obtained in relatively short order. Specific numerical results were obtained for several types of inert gases in liquid sodium during various liquid pressure and temperature transients. In particular, the cases of a bubble flowing with a liquid through a channel with a linear temperature rise and pressure drop (a normal heated channel), a channel with a linear temperature drop (a heat exchanger), and a channel experiencing a sudden pressure drop with an increasing temperature (a 'blocked' heated channel) were all studied.

The second part will treat the entire species continuity equation vigorously. The resulting equations will be an order of magnitude more complex since the species continuity equation will become a partial differential equation. It may be solved with the other ordinary integrated differential equations and/or with the other conservation equations in their original partial differential form.

Finally, the third part will involve the formulation of a more complete energy equation, including the effect of temperature gradients in the liquid–gas solution. The equations will again be partial differential. It should be noted that the solution of the governing equations developed in parts 2 and 3 will require the formulation of a new numerical computer code significantly different from the previous code [4, 5].

The results of each successive step of the analysis development will provide a more comprehensive model than the previous work or step. The inclusion of the various physical effects will allow more confident application of the analysis to a greater variety of engineering problems. The analysis in all of the various stages will still apply to inertia and heat-transfer-dominated regimes of bubble growth and will preclude the necessity to use more specialized models for individual problems.

## PHYSICAL AND MATHEMATICAL MODELS

Consider a spherical gas bubble of radius $R_0$ which at time $t = 0$ is placed into a liquid–gas solution. Its temperature, pressure, dissolved gas concentration, and the concentration of the gas for a saturated solution are equal to $T_0$, $p_{L0}$, $C_{A0}$, and $C_{As}$, respectively. At time $t > 0$, the liquid pressure and temperature are allowed to vary; this variation will have a considerable effect on the behavior of the bubble due to several factors, among which are the temperature dependence of the gas solubility and the vapor pressure, the changing pressure differences across the bubble–solution interface, etc.

If the center of the gas bubble is assumed to be stationary and taken as the origin of a spherical coordinate system, the mass conservation equation for species $A$ diffusing through a solution of $A$ and $B$ may be written as

$$\frac{\partial C_A}{\partial t} = \frac{\mathscr{D}_{AB}}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial C_A}{\partial r}\right) \tag{1}$$

where the assumptions of [4, 5] have been invoked. The initial and boundary conditions consistent with these assumptions are

$$t = 0:\ C_A = C_{A0} \tag{2}$$

$$r = R:\ C_A = C_{As}(t) \tag{3}$$

$$r \to \infty : C_A \to C_{A0}. \tag{4}$$

A simple transformation of the coordinate system permits the reduction of the problem to one which is well known

$$\frac{\partial \phi}{\partial \tau} = \frac{\partial^2 \phi}{\partial \eta^2} \tag{5}$$

$$\tau = 0:\ \phi(\eta, 0) = 0 \tag{6}$$

$$\eta = 0:\ \phi(0, \tau) = \phi_s(\tau) \tag{7}$$

$$\eta \to \infty : \phi(\eta, \tau) \to 0 \tag{8}$$

where

$$\left.\begin{aligned} \eta &= \frac{r}{R} - 1, \tau = \mathscr{D}_{AB}t/R^2, \phi(\eta, \tau) \\ &= \frac{r}{RC_{A0}}[C_A(r, t) - C_{A0}] \\ &\text{and} \\ \phi_s(\tau) &= \frac{C_{As}(t) - C_{A0}}{C_{A0}} \end{aligned}\right\} \tag{9}$$

and has the solution which can be expressed in either of the following forms

$$\phi(\eta, \tau) = \frac{2}{\sqrt{\pi}}\int_{\eta/2\sqrt{\tau}}^{\infty} \phi_s\left(\tau - \frac{\eta^2}{4\xi^2}\right) e^{-\xi^2}\,\mathrm{d}\xi \tag{10a}$$

or

$$\phi(\eta, \tau) = \frac{\eta}{2\sqrt{\pi}}\int_0^{\tau} \frac{\phi_s(\tau - \lambda) e^{-\eta^2/4\lambda}}{\lambda^{3/2}}. \tag{10b}$$

It should be noted that equations (10a) and (10b) do not by themselves specify the concentration field; the saturation function is dependent upon the liquid temperature and the gas partial pressure in the bubble. Thus, additional relationships must be found to completely specify the problem. One such relationship can be obtained from an overall mass balance on the bubble, noting that the rate of change of mass of gas in the bubble equals the rate of transport of gas through the bubble–liquid interface, or

$$-\frac{\mathrm{d}m_A}{\mathrm{d}t} = -4\pi R^2 \mathscr{D}_{AB}\left(\frac{\partial C_A}{\partial r}\right)_{r=R}. \tag{11}$$

Assuming that the gas in the bubble behaves ideally permits $m_A$ to be expressed as

$$m_A = \frac{4\pi M_A p_A R^3}{3R'T} \tag{12}$$

where the temperature of the gas has been taken equal to the liquid temperature. The radial gradient of $C_A$ can be obtained from equations (9) and (10). It is convenient to use equation (10b) for that purpose. Let

$$I = \int_0^\tau \frac{\phi_s(\tau - \lambda)e^{\eta^2/4\lambda}}{\lambda^{3/2}}$$

Then

$$\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0} = \left(\frac{I}{2\sqrt{\pi}}\right)_{\eta=0}$$

and using integration by parts

$$(I)_{\eta=0} = -2\left[\frac{\phi_s(0)}{\sqrt{\tau}} + \int_0^\tau \frac{\phi_s'(\tau - \lambda)}{\sqrt{\lambda}}\mathrm{d}\lambda\right]$$

or in terms of physical variables, the radial gradient of $C_A$ at the bubble surface becomes

$$\left(\frac{\partial C_A}{\partial r}\right)_{r=R} = -\left[\frac{C_{As}(t) - C_{A0}}{R} + \frac{C_{As}(0) - C_{A0}}{\sqrt{(\pi \mathscr{D}_{AB} t)}} + \frac{J}{R\sqrt{(\pi \mathscr{D}_{AB})}}\right] \quad (13)$$

where

$$J = \int_0^{t/R^2} \frac{C_{As}'(\tau)}{(t/R^2 - \tau)^{1/2}}\mathrm{d}\tau$$

and in which $C_{As}'(\tau)$ represents the derivative of $C_{As}$ with respect to $\tau$. The integral $J$ was not included in the analysis of [4, 5] on the grounds of physical reasoning. Its probable importance in such diffusional processes as the bubble growth/collapse in time-dependent pressure fields has been pointed out by Cha and Henry [9].

Prior to substituting equation (13) into (11), the saturation concentration, $C_{As}(t)$, can be related to the partial pressure of the gas in the bubble, $p_A(t)$, by the assumption of mass equilibrium at the bubble–liquid interface. The equilibrium condition, as expressed by Henry's law, may be expressed as

$$C_{As}(t) = \left(\frac{\rho_B M_A}{M_B}\right) K_H[T(t)] p_A(t). \quad (14)^*$$

If it is further assumed that far from the bubble there exists a gas phase with a gas partial pressure of $p_{A\infty}$, and a temperature of $T_\infty$, and initially, the liquid solution is in mass equilibrium with this phase, then the concentration $C_{A0}$ may be expressed as

$$C_{A0} = \left(\frac{\rho_B M_A}{M_B}\right) K_H(T_\infty) p_{A\infty}. \quad (15)$$

Therefore, the overall mass balance, equation (11), can finally be written as follows, using equations (12)–(15)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{p_A R^3}{T}\right) = -\left[\frac{3R'\rho_B \mathscr{D}_{AB} K_H(T_\infty) p_{A\infty}}{M_B}\right] \times \left\{R\left[\frac{K_H(T)p_A}{K_H(T_\infty)p_{A\infty}} - 1\right] + \frac{R^2}{\sqrt{(\pi \mathscr{D}_{AB} t)}}\left[\frac{K_H(T_0)p_A(0)}{K_H(T_\infty)p_{A\infty}} - 1\right]\right\} - \frac{3R'}{M_A}\sqrt{\left(\frac{\mathscr{D}_{AB}}{\pi}\right)} RJ. \quad (16)$$

This equation gives one relationship between the bubble radius, $R$, and the gas pressure in the bubble, $p_A$. The second required relation can be obtained from the continuity and momentum conservation equations for the liquid solution. For an incompressible liquid with no body forces or external temperature effects, it can be shown [12] that these conservation equations can be reduced to the form

$$R\frac{\mathrm{d}^2 R}{\mathrm{d}t^2} + \frac{3}{2}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 = \frac{1}{\rho_B}\left[p_A + p_v - p_L - \frac{2\sigma}{R}\right]. \quad (17)$$

Finally, initial conditions of $p_A$ and $R$ must be specified. By assuming that the initial state of the bubble is one that is in mechanical equilibrium with the liquid solution, it follows that

$$p_A(0) = p_L(0) - p_v(T_0) + \frac{2\sigma(T_0)}{R_0} \quad (18)$$

and

$$\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)_{t=0} = 0. \quad (19)$$

Coupled with the statement that

$$R(0) = R_0 \quad (20)$$

the problem is completely defined by equations (16)–(20), with the functions $K_H(T)$, $p_v(T)$, $\sigma(T)$, and $p_L(t)$ arbitrarily specified.

Prior to a consideration of specific transients in liquid pressure, $p_L(t)$, and temperature, $T(t)$, the defining equations will be cast into a non-dimensional form for ease in subsequent numerical calculations. If the following definitions are applied

$$\tau = \frac{\mathscr{D}_{AB} t}{R_0^2}, \quad \Omega(t) = \frac{R(t)}{R_0},$$
$$\Pi(t) = \frac{p_A(t)}{p_{A\infty}}, \quad \theta(\tau) = \left[\frac{\rho_B R' K_H(T_\infty)}{M_B}\right] T(t) \quad (21)$$

the governing equations, (16)–(20), with some manipulation, become

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{\Pi\Omega^3}{\theta}\right) = -3\Omega[K(\tau)\Pi - 1] - \frac{3\Omega^2}{\sqrt{\pi\tau}}[K(0)\Pi_0 - 1] - 3\gamma\Omega J \quad (22)$$

*This form was chosen since the function $K_H(T)$ was measured as $x_A/p_A$ in [10] and [11], and had the form $\log K_H = a + b/T$.

$$\Omega\frac{d^2\Omega}{d\tau^2}+\frac{3}{2}\left(\frac{d\Omega}{d\tau}\right)^2=\beta\left(\Pi+\Pi_v-\Pi_L-\frac{S}{\Omega}\right) \quad (23)$$

$$\tau=0:\Omega(0)=1 \quad (24)$$

$$\frac{d\Omega}{d\tau}=0 \quad (25)$$

$$\Pi(0)=\Pi_L(0)-\Pi_v(0)+S(0)\equiv\Pi_0 \quad (26)$$

where

$$K(\tau)=K_H[T(t)]/K_H(T_\infty)$$

$$\left.\begin{aligned}\Pi_v(\tau)&=p_v[T(t)]/p_{A\infty}\\ \Pi_L(\tau)&=p_L(t)/p_{A\infty}\\ S(\tau)&=2\sigma[T(t)]/p_{A\infty}\end{aligned}\right\} \quad (27)$$

$$\beta=R_0^2p_{A\infty}/\rho_B\mathscr{D}_{AB}^2,$$

$$\gamma=\frac{R'R_0}{M_A}\sqrt{\left(\frac{\mathscr{D}_{AB}}{\pi}\right)}.$$

An examination of the several terms in equation (22) reveals the three coupled processes that act to change the dimensionless inert gas partial pressure in the bubble. The first term on the right-hand side of this equation represents the resultant decrease in gas pressure with expansion of the bubble; the second term represents the increase in gas pressure with gas temperature; and the third term represents the change in gas pressure associated with the transport of gas between the bubble and the liquid solution. A number of interesting possibilities may be inferred from equations (22) and (23); mostly arising from the non-isothermal condition. For example, it may be possible for inert gas to diffuse out from the bubble, yet to still have significant bubble growth if the rate of rise of temperature is sufficiently rapid. The reverse also holds. Changes in the liquid pressure can also override diffusional effects.

In this paper, several separate cases of liquid temperature and pressure variations will be considered, covering the most commonly found in liquid metal circuits.

*Case 1. Gas bubble in a heat exchanger*

As discussed by Holtz [13], a liquid metal in a circuit can become supersaturated with respect to dissolved gas as the liquid temperature is decreased, if the gas blanket–liquid interface is in a relatively hot section of the circuit. The heat exchanger of many designs of liquid metal loops and liquid metal cooled reactors provide just such a decreasing liquid temperature condition. Assuming that suitable nucleation sites exist in the heat exchanger, it is quite probable that inert gas bubbles will be formed; the actual conditions of temperature and concentration required for this formation have been discussed in [13]. For the purpose of this paper, it will be assumed that bubbles do form in a heat exchanger, and the history of these bubbles as they travel through this decreasing temperature field at essentially constant liquid pressure will be examined.

The one-dimensional liquid temperature profile in a uniformly cooled channel can be easily obtained from an energy balance as

$$T_L(x)=T_L(0)-\frac{4q_{Hx}x}{\rho_Buc_pD_{Hx}}. \quad (28)$$

If the bubble travels with the liquid at the velocity $u$ (i.e. no interfacial slip), the bubble experiences a temperature drop equivalent to

$$T(t)=T_{Hx,0}-(4q_{Hx}/\rho_Bc_pD_{Hx})t. \quad (29)$$

Assuming that the liquid pressure remains constant at a value of $p_{Hx}$, the non-dimensional temperature and pressure functions [from equations (21) and (27)] become

$$\theta(\tau)=\theta_{Hx,0}-\gamma_{Hx,T}\tau \quad (30)$$

$$\Pi_L(\tau)=\Pi_{Hx} \quad (31)$$

where

$$\left.\begin{aligned}\theta_{Hx,0}&=(\rho_B/M_B)R'K_H(T_\infty)T_{Hx,0}\\ \gamma_{Hx,T}&=4q_{Hx}R'R_0^2K_H(T_\infty)/M_Bc_pD_{Hx}\mathscr{D}_{AB}\\ \Pi_{Hx}&=p_{Hx}/p_{A\infty}.\end{aligned}\right\} \quad (32)$$

*Case 2. Gas bubble in a heated channel with liquid pressure drop*

In this case, the temperature and liquid pressure variation associated with the flow of a liquid with an entrained gas bubble through a heated channel will be considered. As in the previous case, a one-dimensional approximation will be made, resulting in the following dimensionless temperature and liquid pressure functions

$$\theta(\tau)=\theta_0+\gamma_T\tau \quad (33)$$

$$\Pi_L(\tau)=\Pi_{L0}-\gamma_p\tau \quad (34)$$

where the symbols are defined in the Nomenclature.

*Case 3. Heated channel with sudden flow blockage*

A situation that is of some interest in the safety analyses of liquid metal cooled, fast breeder reactors is that of an instantaneous, complete blockage of a coolant flow channel with continued heating. In this case, at the instant of blockage, the liquid pressure will drop to the value in the gas plenum (plus the liquid head), and the static liquid will continue to be heated. Thus, the temperature and pressure field experienced by a bubble that enters the channel at time zero and reaches the end of the heated section at the instant of flow blockage (at time $t_B=L/u$) will be

$$T(t)=\begin{cases}T(0)+\dfrac{4q_wt}{\rho c_pD_H} & 0<t<t_B\\[2ex] T(t_B)+\dfrac{4q_wt}{\rho c_pD_H} & t>t_B\end{cases} \quad (35)$$

$$p_L(t) = \begin{cases} p_L(0) - \dfrac{u\Delta pt}{L} & 0 < t < t_B \\ p_{L,b} & t > t_B. \end{cases} \quad (36)$$

Equation (35) may be simply rewritten as

$$T(t) = T(0) + \frac{4q_w t}{\rho c_p D_H} \quad t > 0. \quad (37)$$

In terms of the dimensionless parameters, equations (37) and (36) become

$$\theta(\tau) = \theta_0 + \gamma_T \tau \quad \tau > 0 \quad (38)$$

$$\Pi_L(\tau) = \begin{cases} \Pi_{L0} - \gamma_p \tau & 0 < \tau < \tau_B \\ \Pi_{L,b} & \tau > \tau_B \end{cases} \quad (39)$$

where the symbols are defined in the Nomenclature.

## APPLICATIONS AND DISCUSSION OF RESULTS

The rate of discharge of water from the accumulator of the emergency core cooling system (ECCS) of a light water reactor under abnormal reactor conditions may be affected in various degrees by the presence of inert gas bubbles in the water. The bubbles are produced as a result of the depressurization of the accumulator which is initially saturated with nitrogen gas. The effect of the bubbles flowing with the liquid through the nozzle into the core is to decrease the rate of supply of coolant. The theory presented in this paper will provide an effective tool for analyzing this problem and related experimental investigations.

It is difficult in many instances to predetermine the magnitudes of all of the physical phenomena that would affect the bubble dynamics in any given problem. The case of transients further complicates this situation, especially as the rate of the transient becomes large. Such is the case of the ECCS blowdown when the water pressure in the accumulator is rapidly reduced by approximately 4 MPa. It was noted by Cha and Henry [14] that bubble growth rates estimated from constant pressure predictions differed drastically from experimental results obtained during a fast depressurization. The effects of other physical parameters are equally difficult to estimate. Thus, an extensive analysis was required to gain confidence that an important effect has not been neglected. The theory presented in this paper will provide such an analysis.

The study of bubble dynamics has important application to liquid metal fast breeder reactor (LMFBR) normal and abnormal operation [4–7]. Argon gas bubbles appear in the sodium as a result of cooling the hot plenum sodium, saturated with argon gas, in the intermediate heat exchanger (IHX). The growth or collapse of these bubbles in the reactor core during rapid transient conditions is an important consideration affecting the incipient boiling superheat of the sodium and the related safety implications [13]. Considerable experimental and analytical effort has been given to this problem. The analysis of [4–7] was developed particularly for it and the results to date indicate a safe situation. This paper extends that analysis and provides a more comprehensive analytical basis in support of this important work.

Last but not least, it should be pointed out that the general theory presented here could very well be used for verifying many other less complete models that have been developed for more specific applications. Recent investigations which might usefully employ the present theory for this purpose include: bubble growth in variable pressure field work of [2], bubble dissolution studies of [15], cavitation studies of [16], and others.

To demonstrate the application of the theory developed in this paper, one must solve equations (21)–(27) for specified liquid temperature and pressure variations. An example chosen was that of gas bubbles traveling in a heated channel with liquid pressure drop, equations (33) and (34), as described under Case 2 in the paper. The above nonlinear system of equations could not be solved analytically; therefore, a numerical integration technique known as the fourth-order Runge–Kutta method was used.

Numerical results were obtained for four combinations of inert-gas–sodium systems — argon–sodium, helium–sodium, krypton–sodium and xenon–sodium, since this is of most practical interest for fast breeder reactors (LMFBR). The importance of the argon–sodium system has been sufficiently well outlined in [4–5]. It should be noted, however, that if the fuel is vented, the primary fission gas in the gas blanket will be krypton. But if the fuel is not vented, the krypton will be converted to various xenon isotopes. Also, as pointed out in [17], the use of helium in place of argon as an inert cover gas is often preferred, when taking into account various design considerations. Thus, the reasons for selecting Ar–Na, He–Na, Kr–Na and Xe–Na combinations for the numerical studies reported here are justified.

Since the numerical calculations require the values for the Henry's coefficient $K_H$ and the values for the diffusion coefficient $\mathscr{D}_{AB}$, references [10] and [11] were used for providing that information. Sodium properties were calculated according to [18]. All results were obtained for the behavior of inert gas bubbles as they pass through an LMFBR operating conditions as summarized in Table 1. These computed results are presented graphically in Figs. 1–3. Figure 1 illustrates the behavior of various initial-sized bubbles as they pass through a heated channel. It is interesting to note

Table 1. Operating conditions

| Parameter | Value |
|---|---|
| $D_H$ | 3.075 mm |
| $L$ | 914.4 mm |
| $p_{A\infty}$ | 101.3 kPa |
| $p_{L\infty}$ | 101.3 kPa |
| $T_0$ | 316 °C |
| $T_\infty$ | 471 °C |
| $u$ | 5.18 m s$^{-1}$ |

that all bubbles (involving various inert gases) will survive the transit through such a channel. Furthermore, the growth rate in all cases is very small.

The variation of the partial pressures of inert gases inside the various-sized bubbles with transit time through the channel is plotted in Fig. 2. One notes by examining that figure that for a given initial-sized bubble, the variation of $p_A$ with time is independent of the kind of inert gas under study and that in all cases $p_A$ decreases with increasing transit time. Furthermore, these variations could be very closely fitted with straight lines, the slopes of which being independent of inert gases or initial radii $R_0$. The only exception is helium, which, for $R_0 = 10\,\mu$m, deviates slightly from the above observed behavior.

Finally, Fig. 3 presents the fractional increases in mass of inert gases inside the bubble as they traverse the heated channel, for $R_0 = 1000\,\mu$m. As expected by reference to Fig. 1, all curves rise with time and with approximately the same rate.

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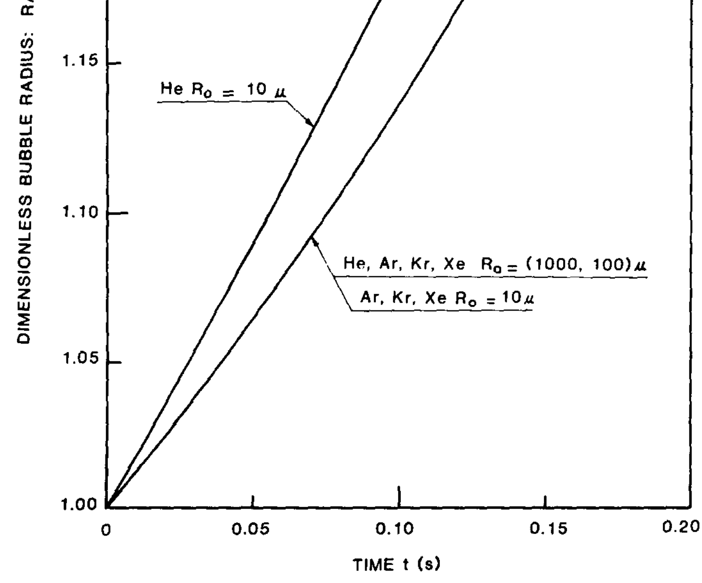


Fig. 1. Dimensionless bubble radius results for various inert gases.

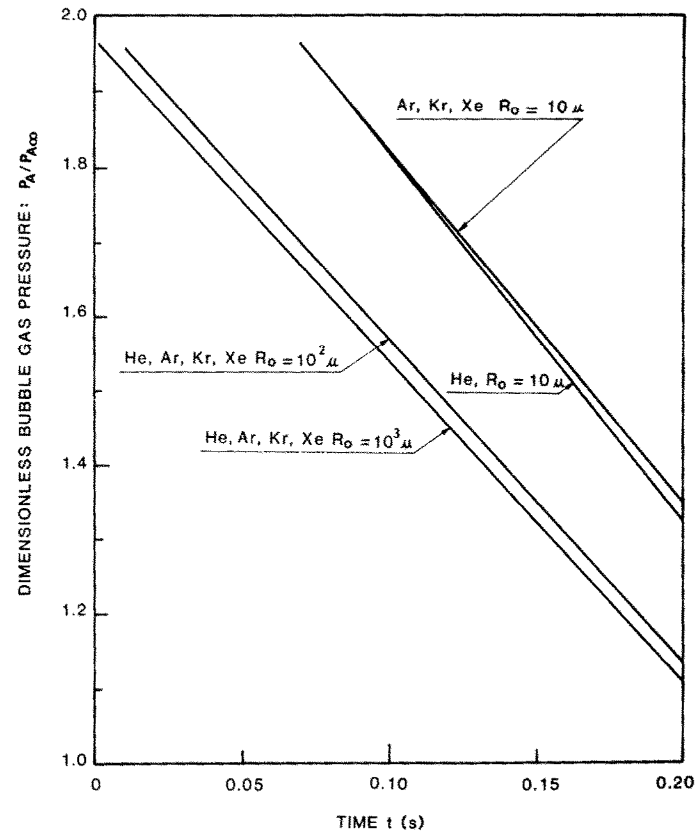


FIG. 2. Dimensionless gas partial pressure in bubble for various inert gases.

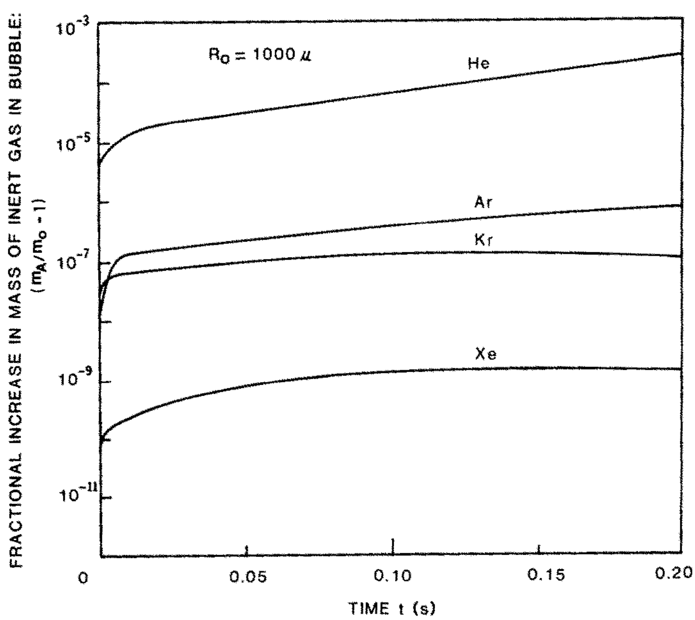


FIG. 3. Results for gas mass fraction in bubble for various inert gases.

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## DYNAMIQUE DES BULLES DE GAZ INERTE DANS DES SYSTEMES DE CONVECTION FORCEE

**Résumé**—Cet article est une extension d'une publication traitant des effets des variations de masse, de quantité de mouvement et d'énergie sur la dynamique des bulles de vapeur–gaz se déplaçant à travers une solution à deux composants de gaz dans un liquide et soumise à des évolutions de pression et de température du liquide. On étudie particulièrement un écoulement de liquide avec bulles dans: un canal avec une élévation linéaire de température et une chute de pression (un canal normalement chauffé); un canal avec une chute de température linéaire (un échangeur de chaleur); un canal subissant une soudaine chute de pression avec un accroissement de température (un canal chauffé et bloqué). Des résultats munériques sont obtenus pour différents types de gaz inertes dans le sodium liquide pour le cas d'un canal normal chauffé.

## DIE DYNAMIK VON INERTGASBLASEN IN SYSTEMEN MIT ERZWUNGENER KONVEKTION

**Zusammenfassung**—Diese Arbeit ist eine Erweiterung einer vorausgegangenen Veröffentlichung über den Einfluß von Massen-, Impuls- und thermischen Änderungen auf die Dynamik von Dampf–Gas–Blasen in einer verdünnten Zweikomponentenlösung des Gases in einer Flüssigkeit bei verschiedenen instationären Druck- und Temperaturverläufen in der Flüssigkeit. Für eine mit einer Flüssigkeit strömende Blase werden insbesondere folgende Fälle untersucht: ein Kanal mit linearem Temperaturanstieg und Druckabfall (normal beheizter Kanal), ein Kanal mit linearem Temperaturabfall (Wärmetauscher) und ein Kanal bei plötzlichem Druckabfall und zunehmender Temperatur ("blockierter" beheizter Kanal). Für verschiedene Inertgas-Typen in flüssigem Natrium werden für den Fall eines normal beheizten Kanals numerische Ergebnisse mitgeteilt.

## ДИНАМИКА ПУЗЫРЬКОВ ИНЕРТНЫХ ГАЗОВ В СИСТЕМАХ С ВЫНУЖДЕННОЙ КОНВЕКЦИЕЙ

**Аннотация** — Предлагаемая статья является продолжением ранее опубликованной работы по исследованию влияния изменения массы, импульса и температуры на динамику паро–газовых пузырьков, движущихся в разбавленном двухкомпонентном растворе газа в жидкости при изменении давления и температуры жидкости. В частности, исследуется движение пузырьков в потоке жидкости в канале при линейном увеличении температуры и уменьшении давления (нагреваемый канал), при линейном снижении температуры (теплообменник) и в канале, в котором происходит внезапное падение давления и постепенное увеличение температуры (нагреваемый канал с явлением «запирания»). Получены численные результаты по течению в нагреваемом канале растворов некоторых видов инертных газов в жидком натрии.